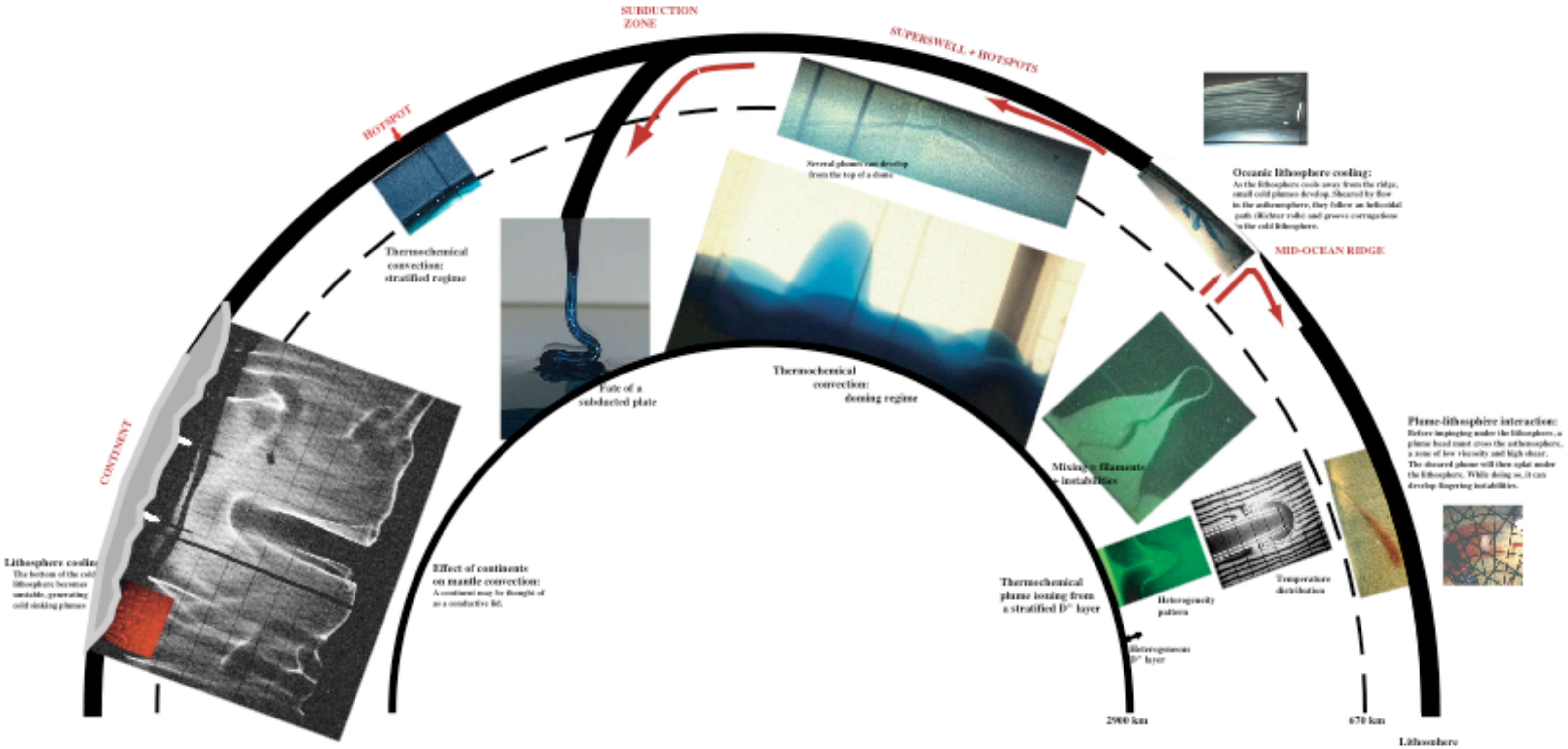
Experimental and Theoretical Studies of Mantle Convection

-Hot instabilities and hot spots
-Plates and subduction

-Scaling

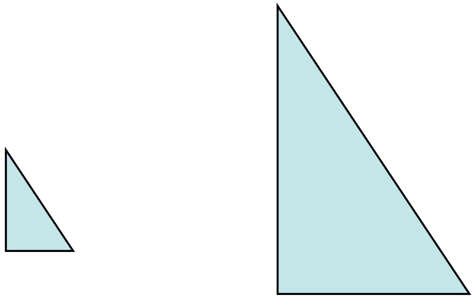
Neil M. Ribe (stepping in for Anne Davaille)



The Concept of « Scaling »

Dynamical Similarity

- **Starting point: ancient Greek idea of geometrical similarity:**



☞ **only *dimensionless* parameters matter** (angles, ratios of sides …)

Galileo's idea (1638): generalize the similarity concept to *mechanical* systems

(ships in drydock; stone columns; animal skeletons)

☞ **What are the appropriate dimensionless quantities for a given mechanical system?**

Dimensionless parameters: How many?

Two rigorous ways to determine this:

1. **Nondimensionalize the governing equations** (if you know them)

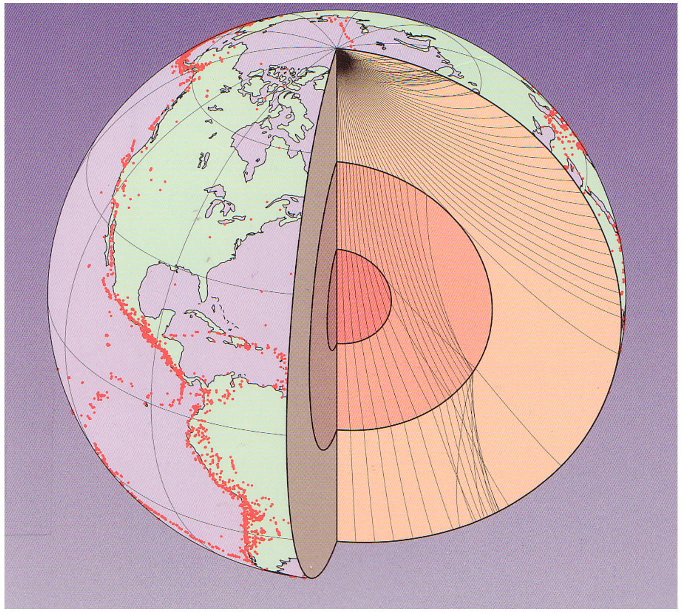
   ☞ **Rewrite the equations in terms of dimensionless (scaled) variables**

2. **Use Buckingham's Π-theorem** (if you don't)

☞ **Number of independent dimensionless groups =**

number of physical parameters - number of these parameters that have independent dimensions

Example: thermal convection with T-dependent viscosity



Boussinesq equations,

$$
\begin{aligned}
\vec{\tilde{\nabla}} \cdot \tilde{v} &= 0, \\
\frac{1}{Pr}\frac{D\vec{\tilde{v}}}{D\tilde{t}} &= -\vec{\tilde{\nabla}}\tilde{P} + \vec{\tilde{\nabla}} \cdot \left(\frac{\mu}{\mu_0} (\vec{\tilde{\nabla}}\vec{\tilde{v}} + [\vec{\tilde{\nabla}}\vec{\tilde{v}}]^t) \right) - \frac{\vec{\bar{g}}}{g_0}\frac{\bar{\alpha}}{\alpha_0} Ra\tilde{T}, \\
\frac{D\tilde{T}}{D\tilde{t}} &= \vec{\tilde{\nabla}} \cdot \left[\frac{\bar{k}}{k_0} \vec{\tilde{\nabla}}\tilde{T} \right] + \frac{1}{Ra}\frac{\rho_0 H a^2}{k_0 \Delta T}.
\end{aligned}
$$

Boundary conditions:
-Free surface
-Constant temperature

3 dimensionless groups:

$Pr = \kappa/\nu$ $(\sim 10^{23})$

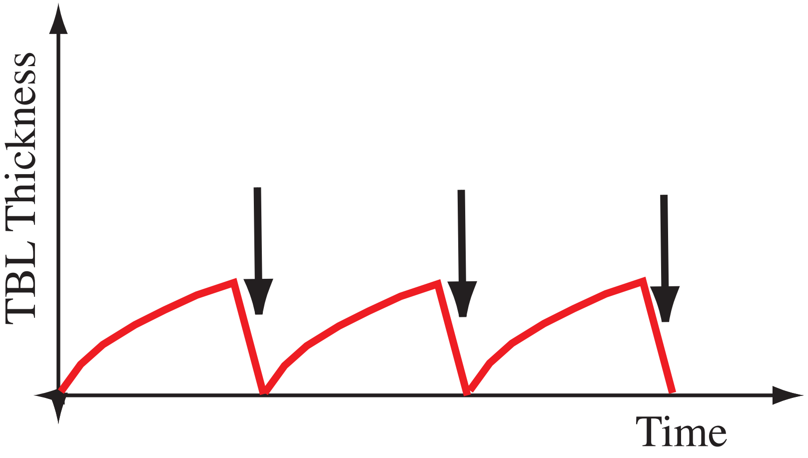
$Ra = \alpha . g . \Delta T . d^3 / (\kappa . \nu)$ $(\sim 10^6 - 10^9)$

$\gamma = \mu_{top}/\mu_{bot}$

What is a scaling law?

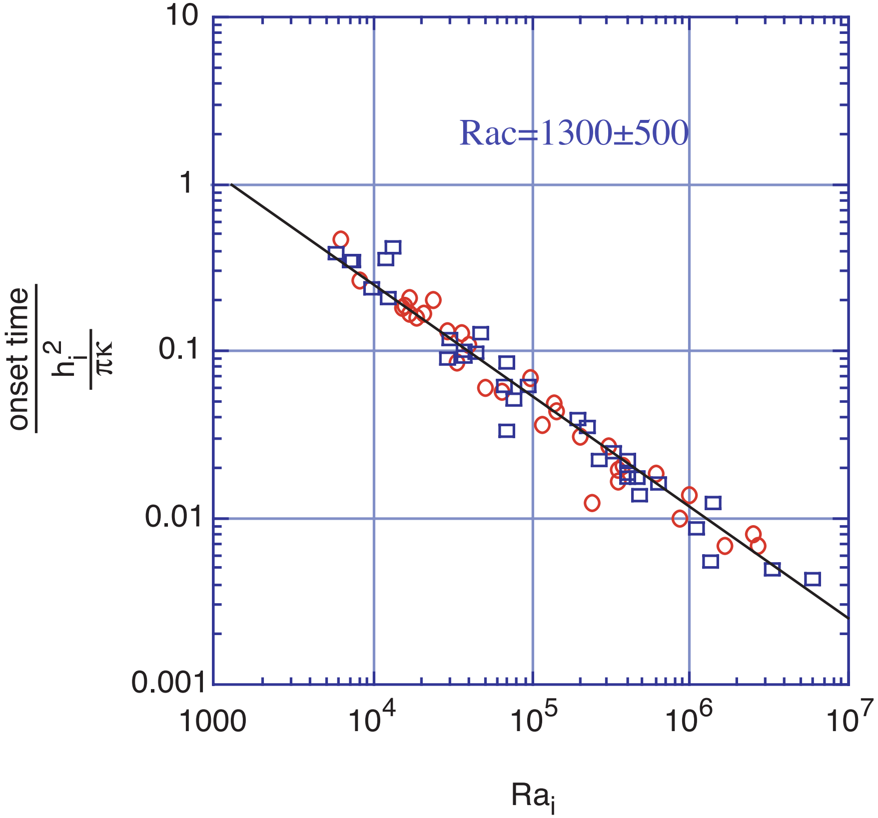
☞ **A quantitative relation** (analytical, numerical or experimental)
between a model « output » parameter of interest
and the « input » parameters that control it.

Example: onset time of convection in
an impulsively heated fluid layer



$$\tau = (Ra_c/Ra)^{2/3}\ .\ d^2/(\kappa)$$

$$Ra = \alpha.g.\Delta T.d^3\ /\ (\kappa.\nu)$$



(Le Bars & Davaille 2004)

Geodynamical modeling: A personal view

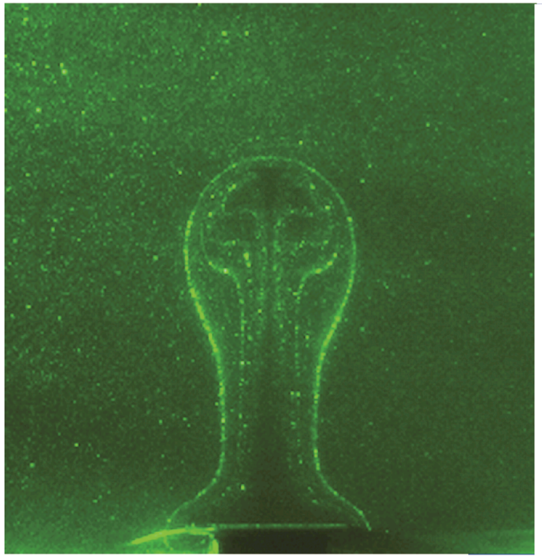
1. Identify the phenomenon of interest (from observations)
2. Formulate a simplified model problem (geometry, boundary-initial conditions, governing equations)
3. Dimensional analysis (identify key control parameters)
4. Build the data base systematically (lab experiments, numerical models)
5. Derive regime diagrams and quantitative scaling laws for key output parameters
6. »Scale up » to the Earth (compare model predictions to observations)

Regimes of Thermal and Thermochemical Convection

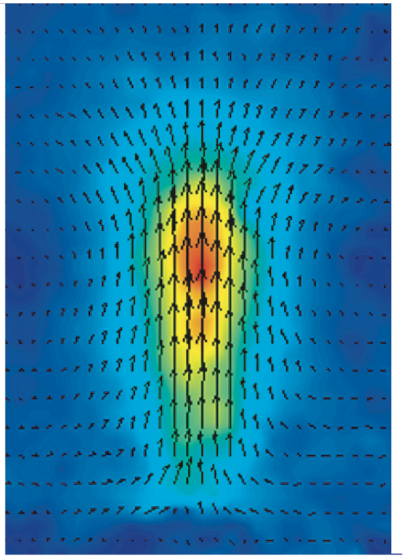
Visualisation:

Simultaneous in situ determination of:

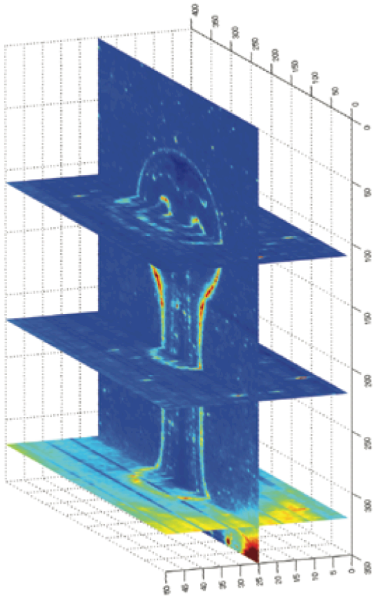
+temperature field + local Tp gradient
(Thermochromic Liquid Crystals, differential interferometry)
+velocity field (PIV)
+concentration field (LIF)



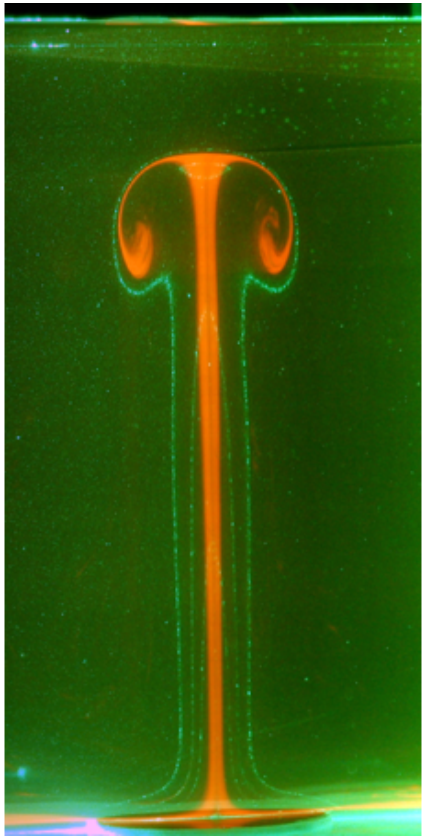
Isotherms
24.4, 27.0, 31.1, 35.0, 39.5 °C



Velocity field (PIV)

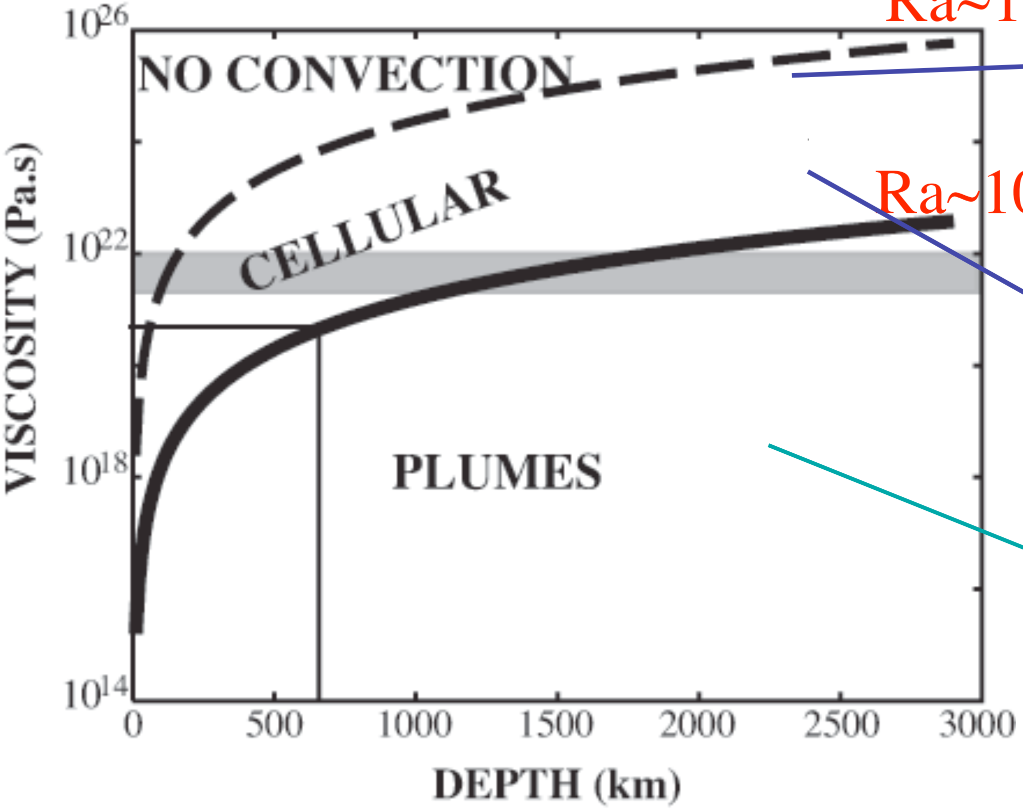


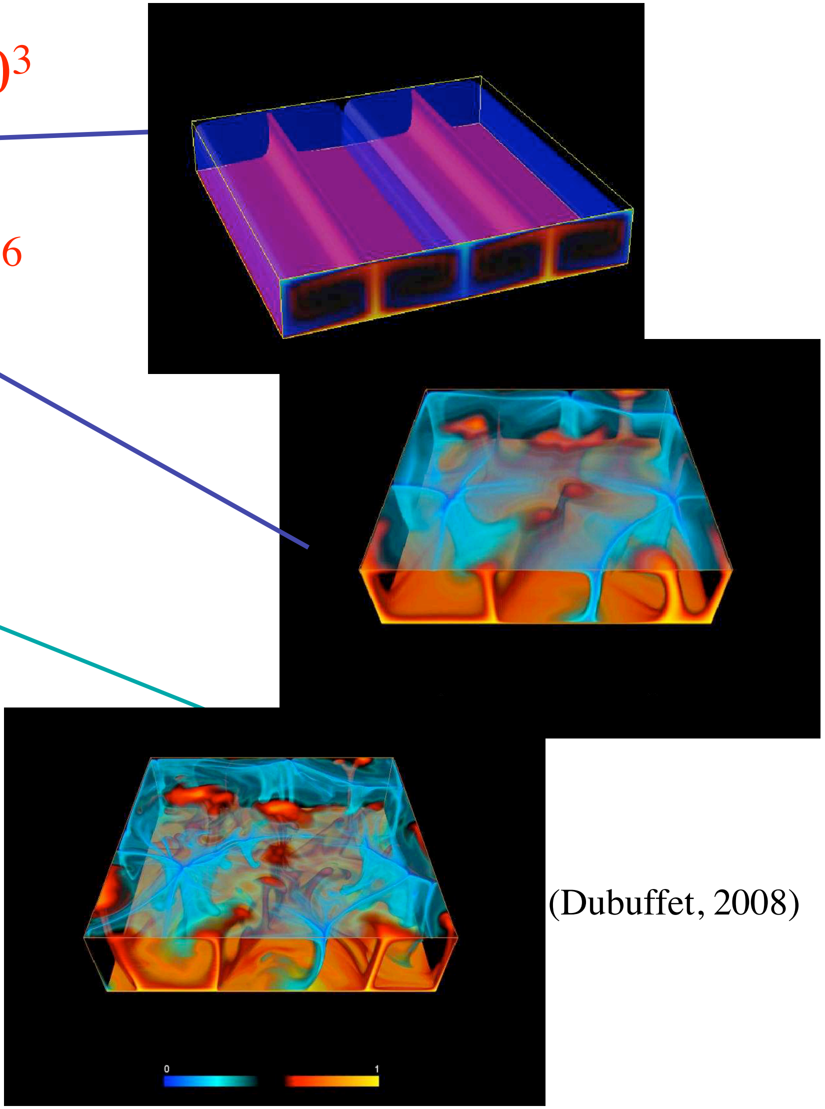
Isotherms 3D



Isotherms + concentration

Regimes of isoviscous convection





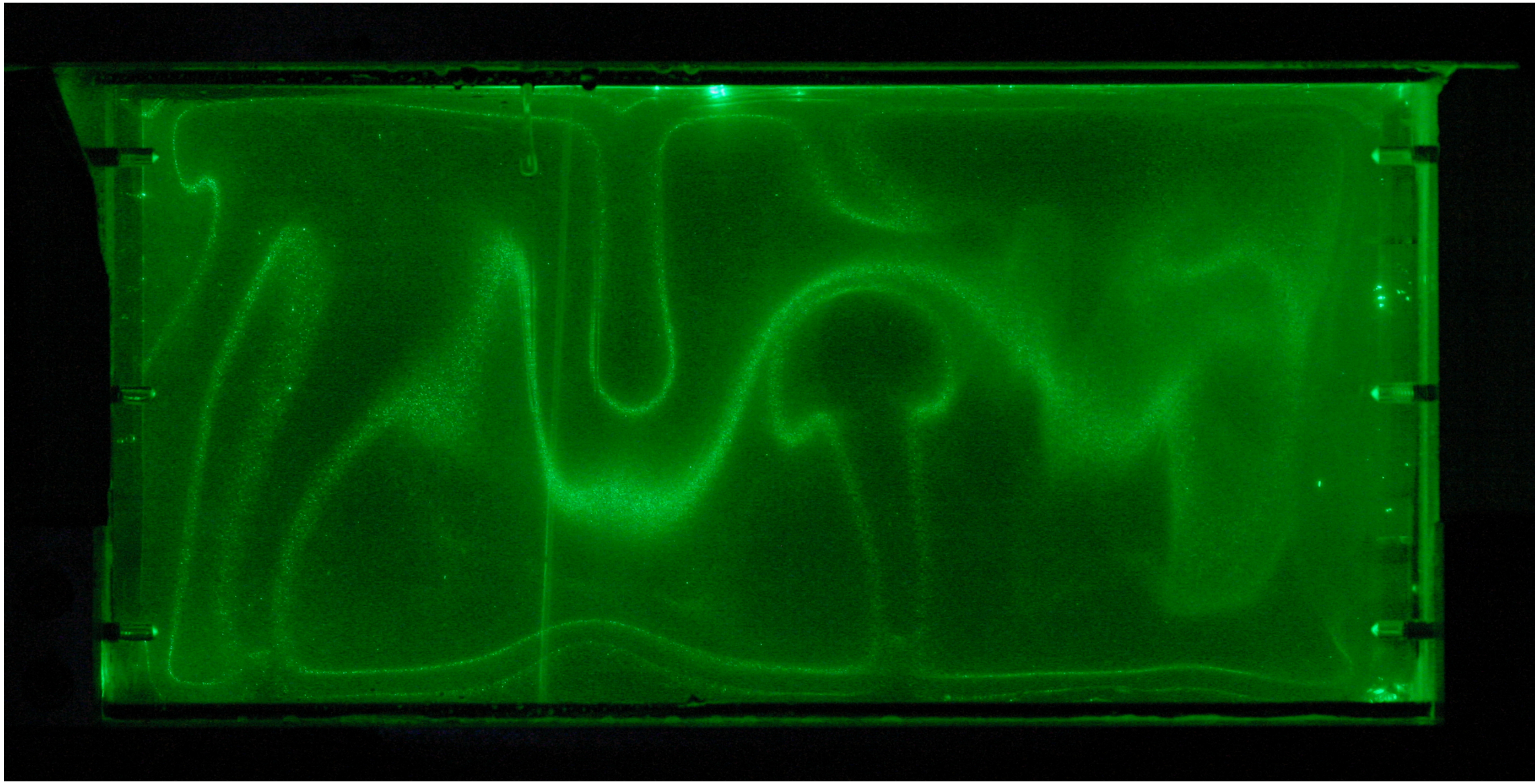
Ra~10^3

Ra~10^6

(Dubuffet, 2008)

$$Ra = \frac{\alpha.g.\Delta T.h^3}{\kappa.\mu/\rho}$$

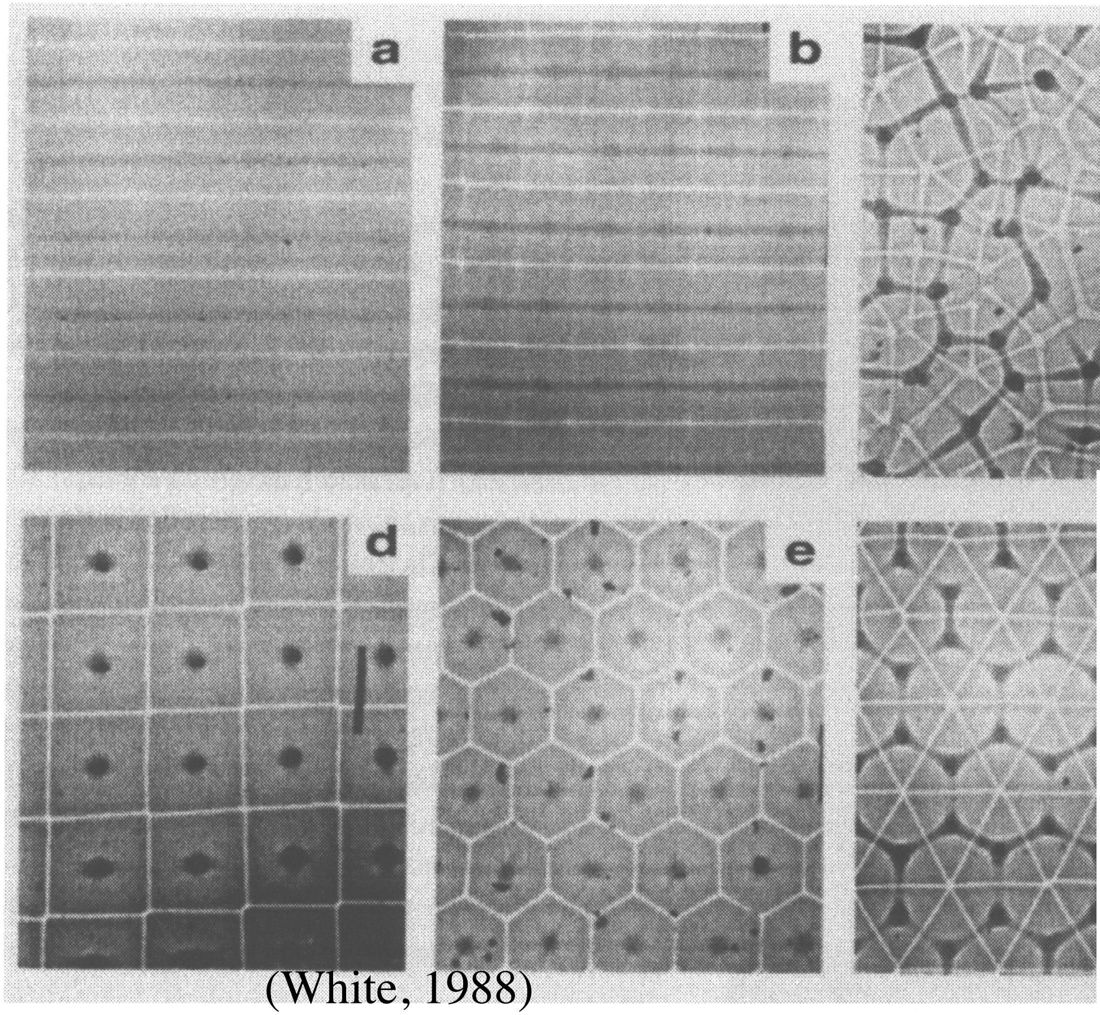
Convection in a fluid with temperature-dependent viscosity: Morphology of upwellings/downwellings



Hot and cold TBL instabilities for sugar syrup cooled from above and heated from below ($Ra=4.7x10^6$) . The viscosity contrast between the coldest (5.0°C) and the hottest fluid (51.9°C) is 116.

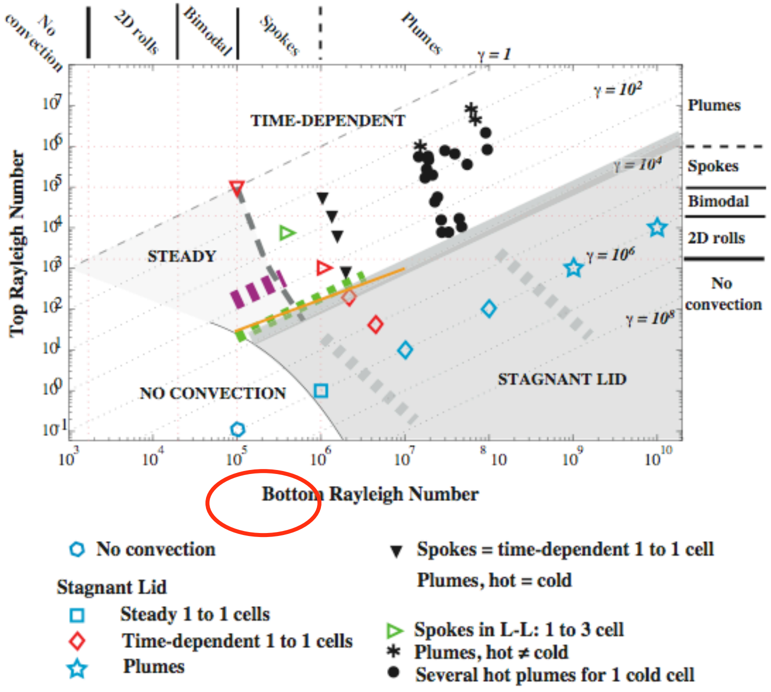
Convection in a fluid with T-dependent viscosity: Planforms and regime diagram

Planforms for small Ra



(White, 1988)

Regime diagram



Thermochemical convection in a two-layer mantle

Four dimensionless groups:

governing stability:

- $Ra = \dfrac{\alpha g \Delta T d^3}{\kappa \nu}$
- $B = \dfrac{\Delta\rho_\chi/\rho}{\alpha \Delta T}$

governing morphology:

- $\gamma = \eta_1/\eta_2$
- $a = d_1/d_2$

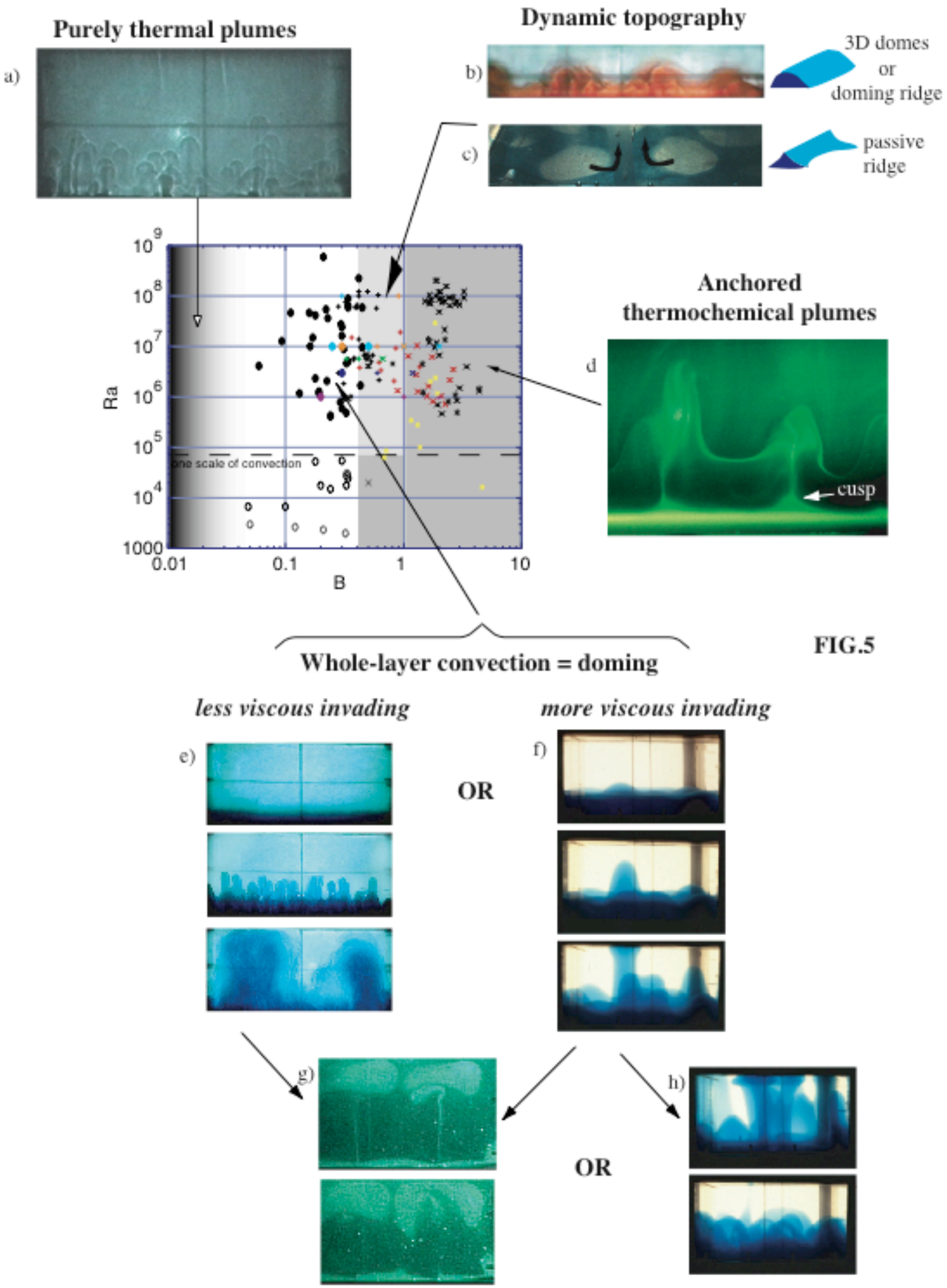
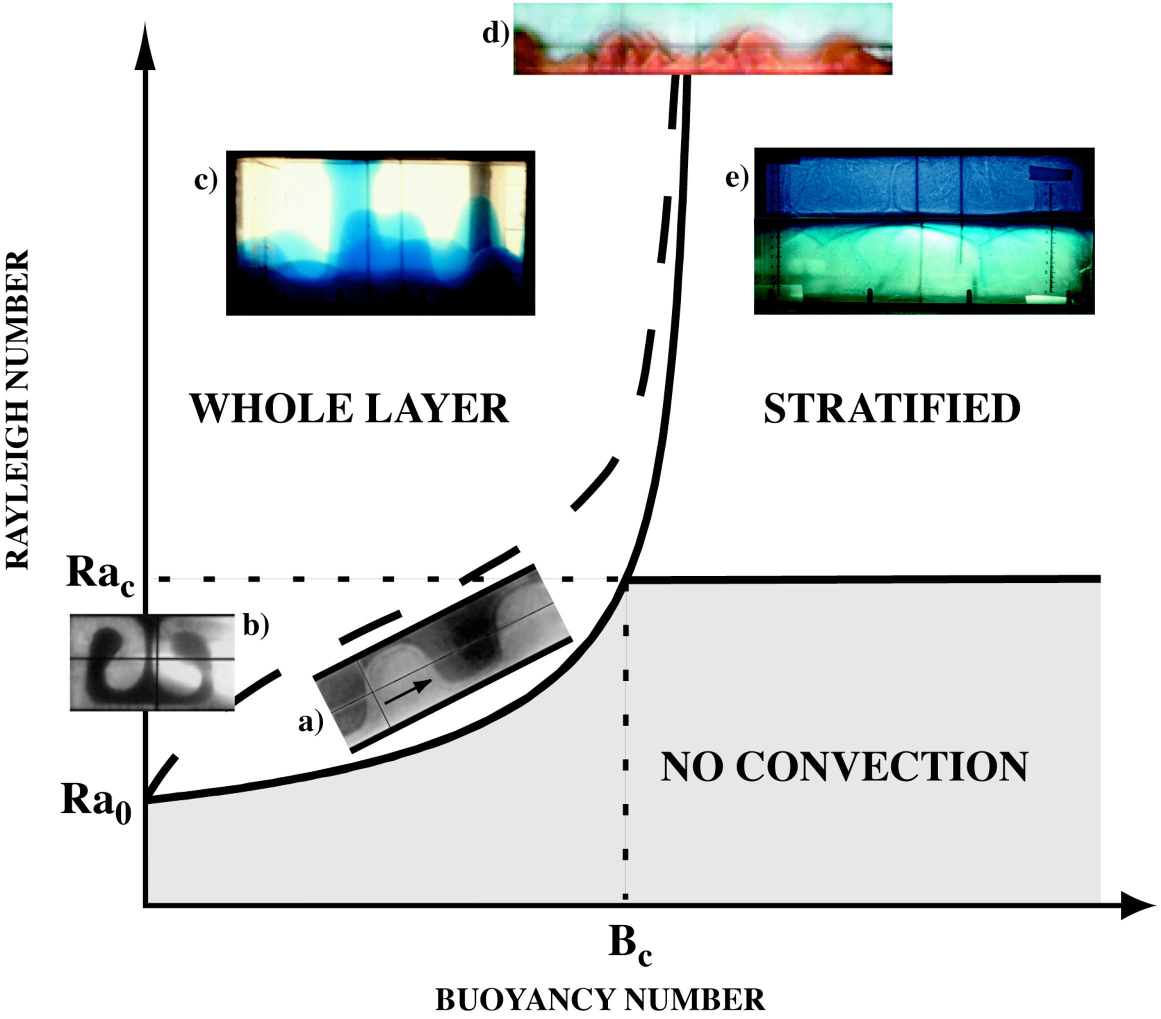


FIG.5

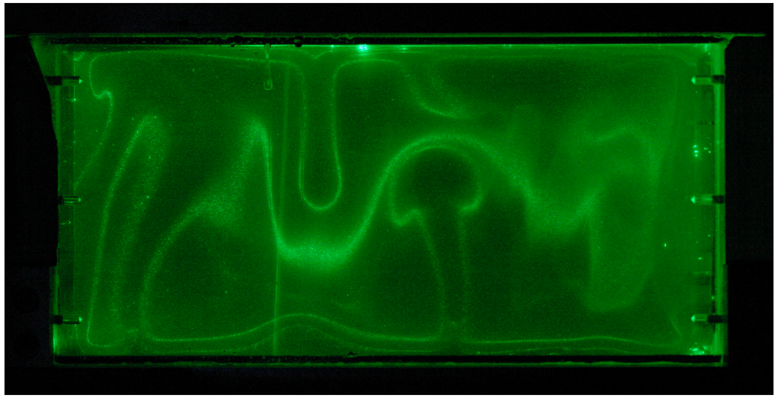
Thermochemical convection : Regime diagram

DIAGRAMME DES RÉGIMES

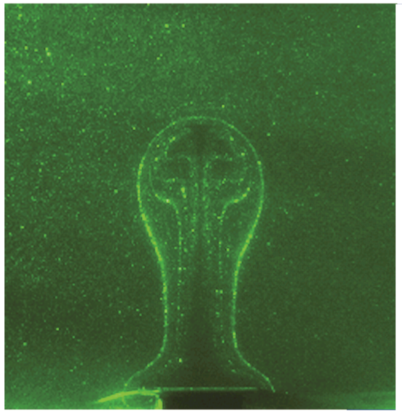


ESR8: Thermal convection with plate tectonics in the laboratory

⇒
- Use non-newtonian fluids (colloids, polymers)
- Study convection in the Rayleigh-Bénard configuration



⇒ Regime diagrams,
characteristics of the flow



Isotherms
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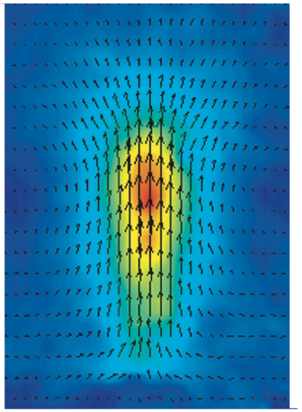
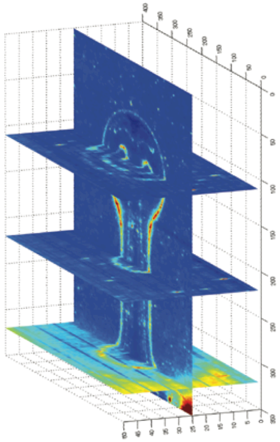


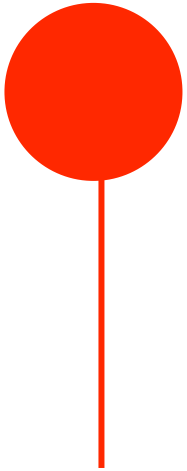
Velocity field (PIV)

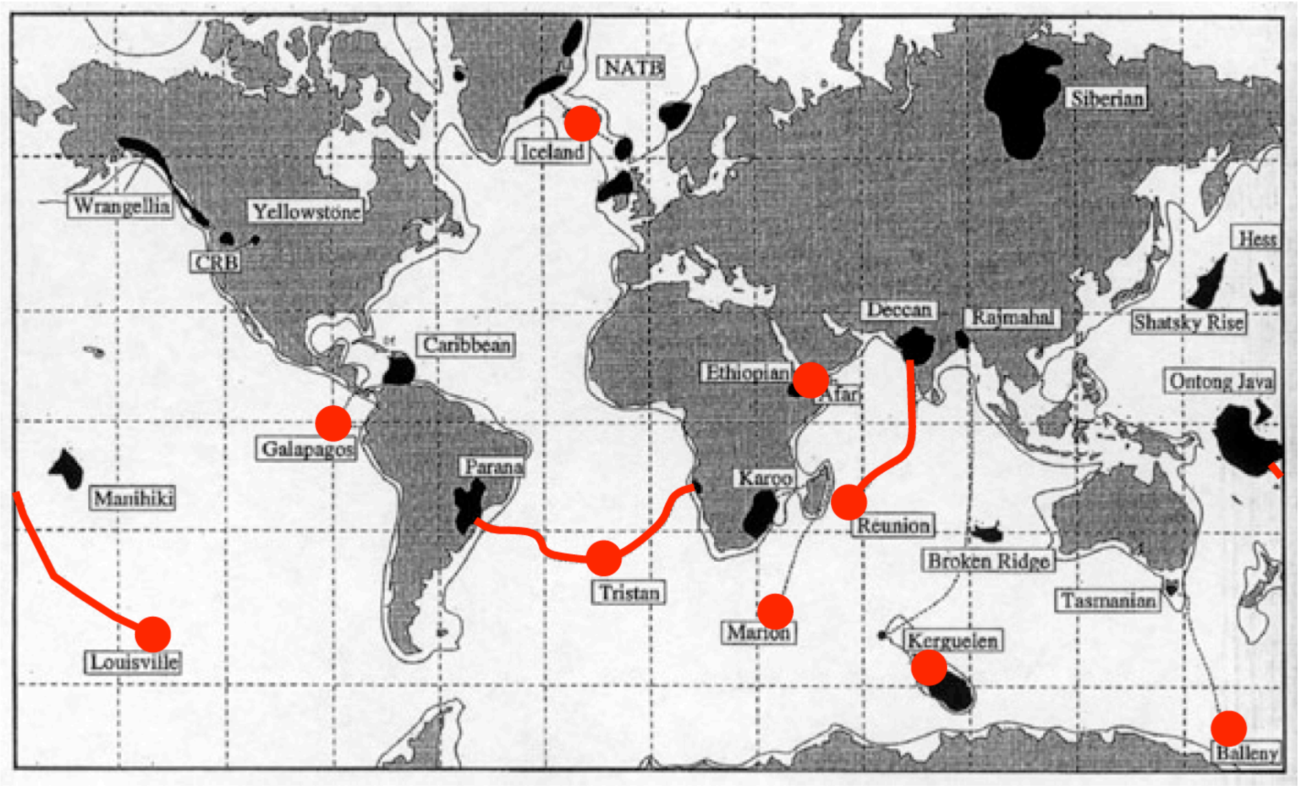


Isotherms 3D

Phenomenology of Thermal Plumes

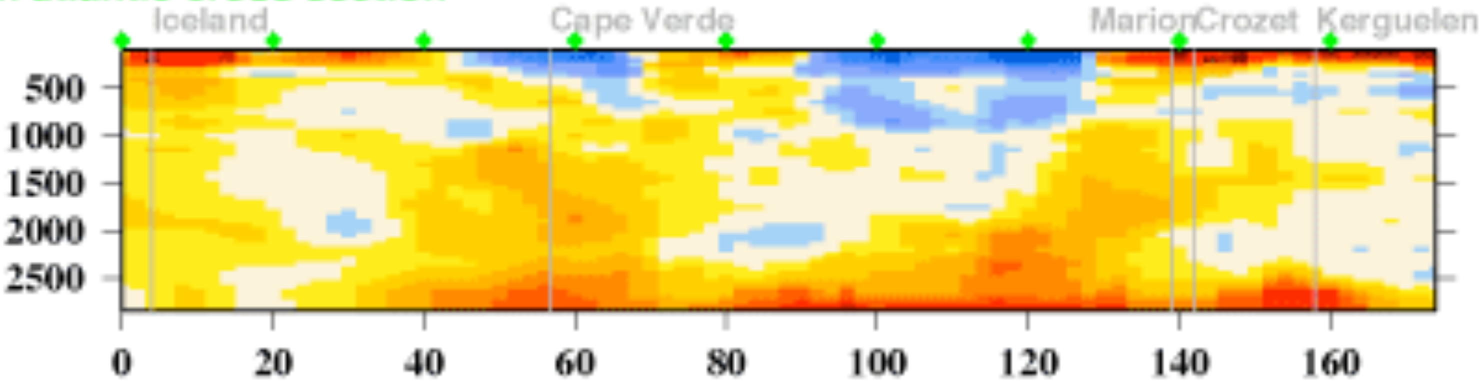
Intraplate volcanism : long track with traps
=> plume = head + stem

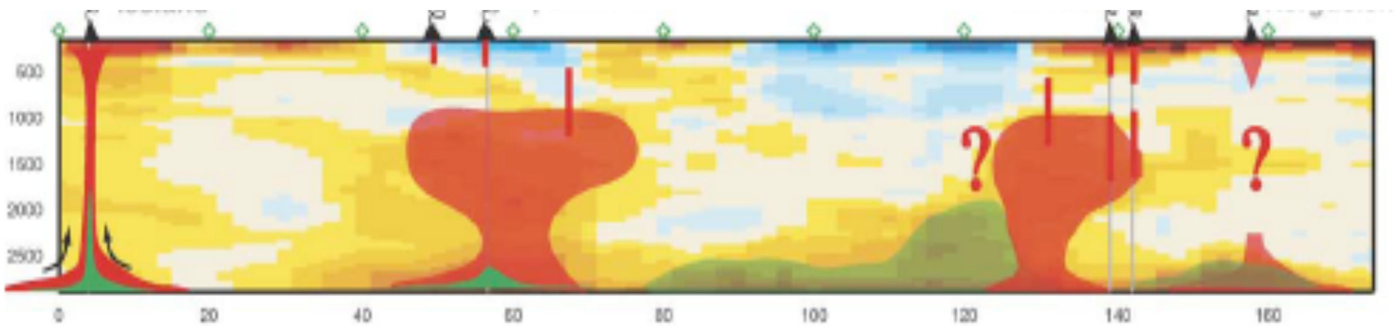


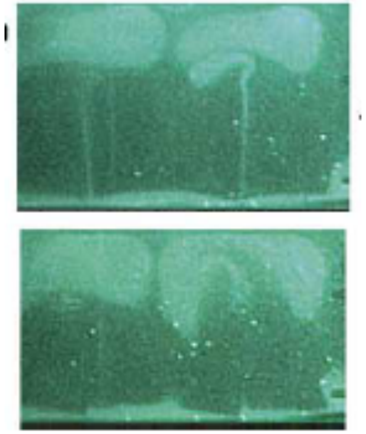


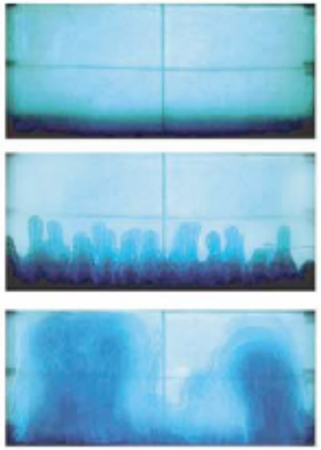
(Richards & al, 1989)

b) Circum atlantic cross section

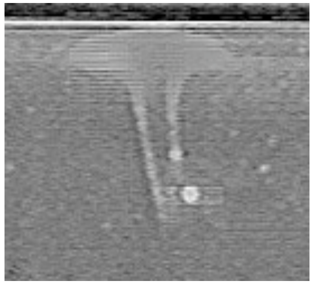


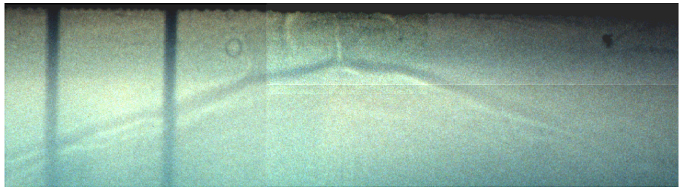




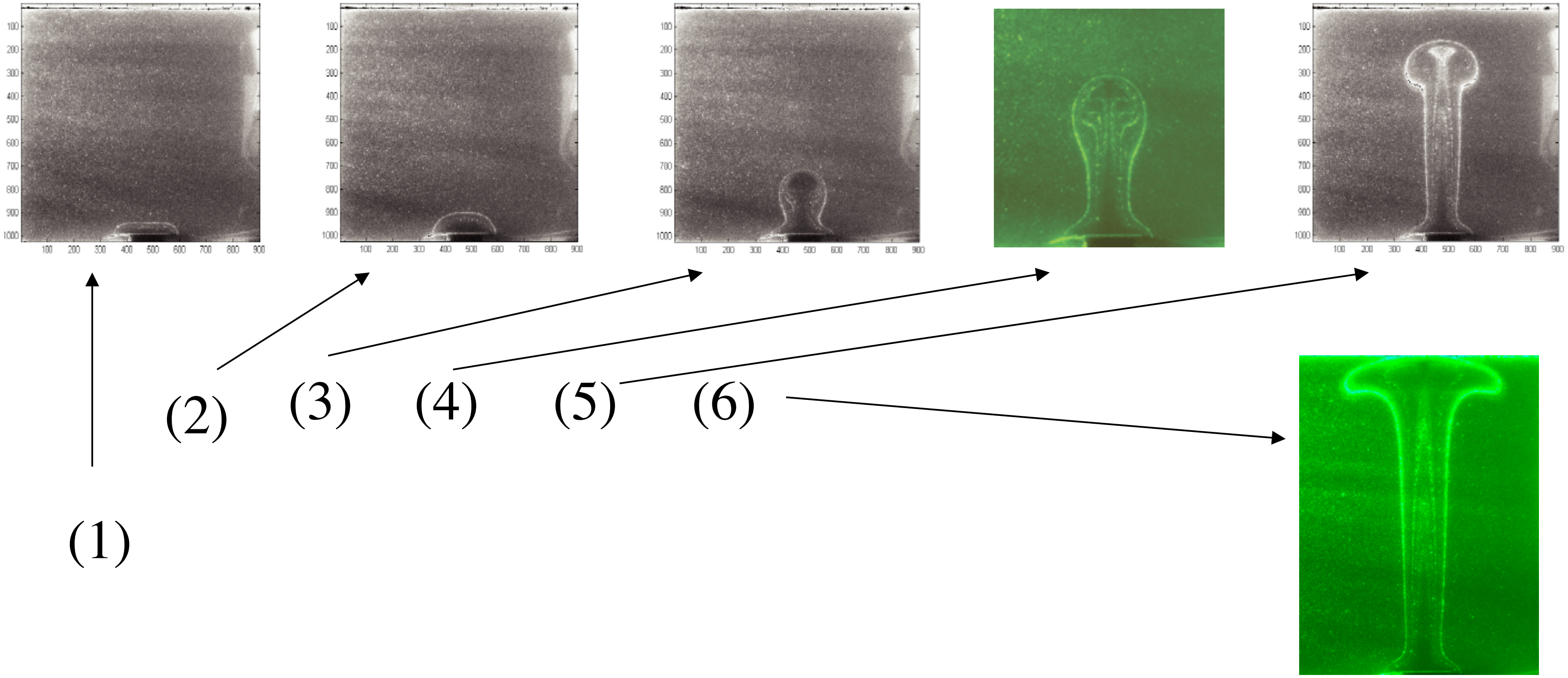


DIFFERENT TYPES
of
HOT SPOTS





Plume from an isolated heat source in an isoviscous fluid



(1)

(2)

(3)

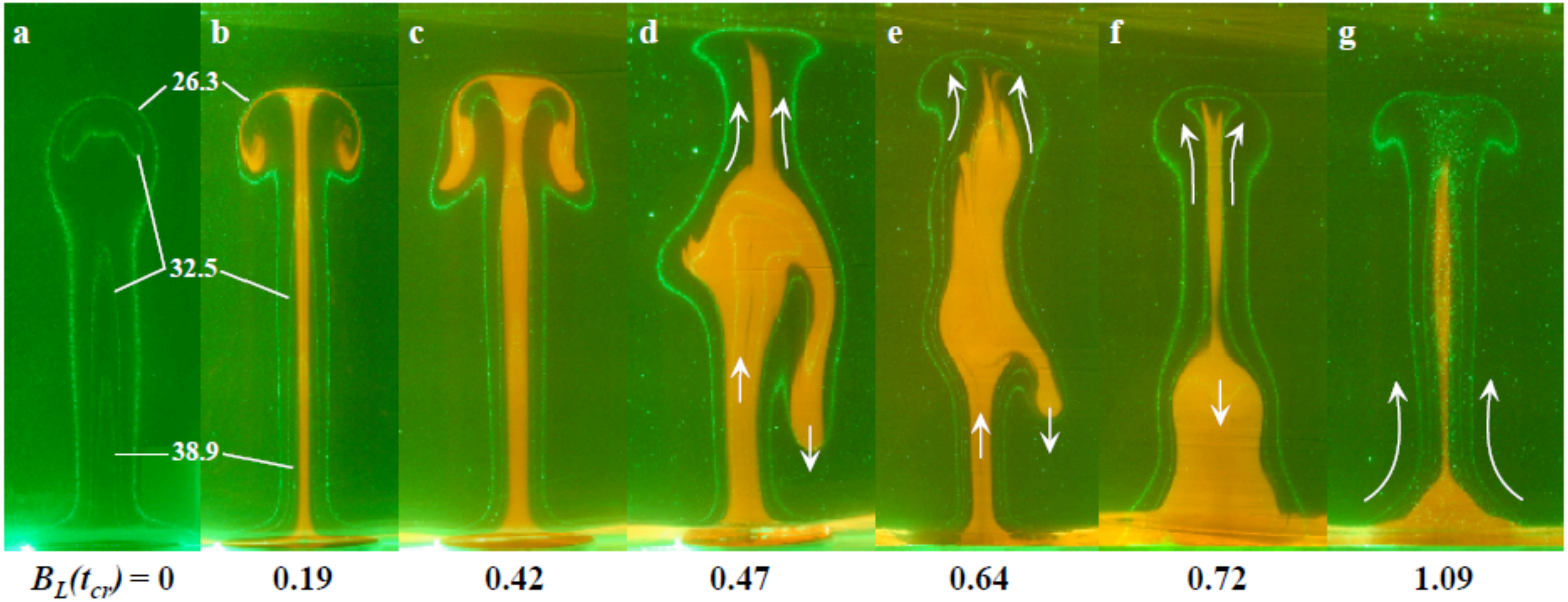
(4)

(5)

(6)

Localized thermochemical plumes

Initial buoyancy ratio

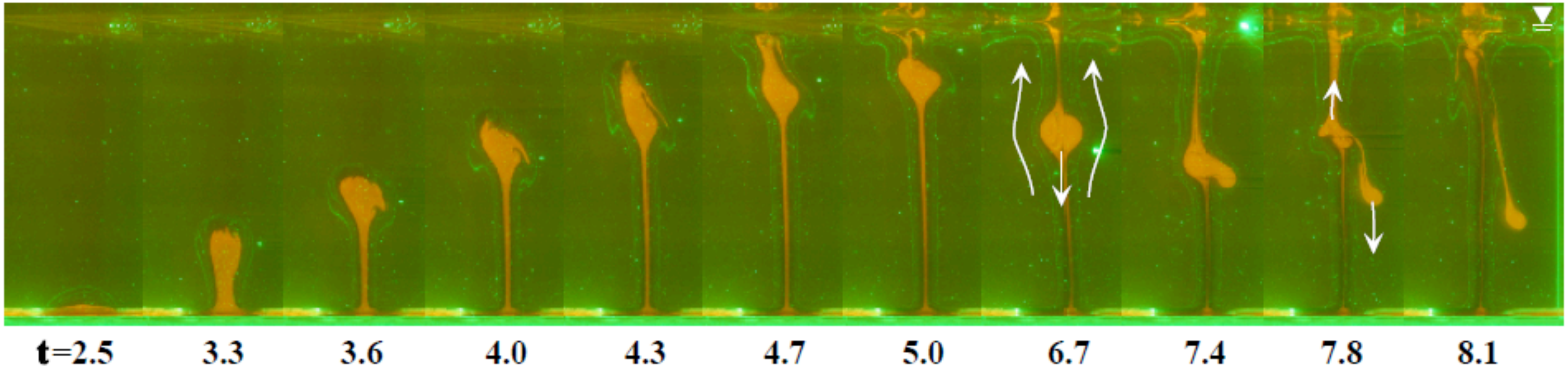


$$B_L = \Delta\rho_x / \rho_0 \alpha \Delta T$$

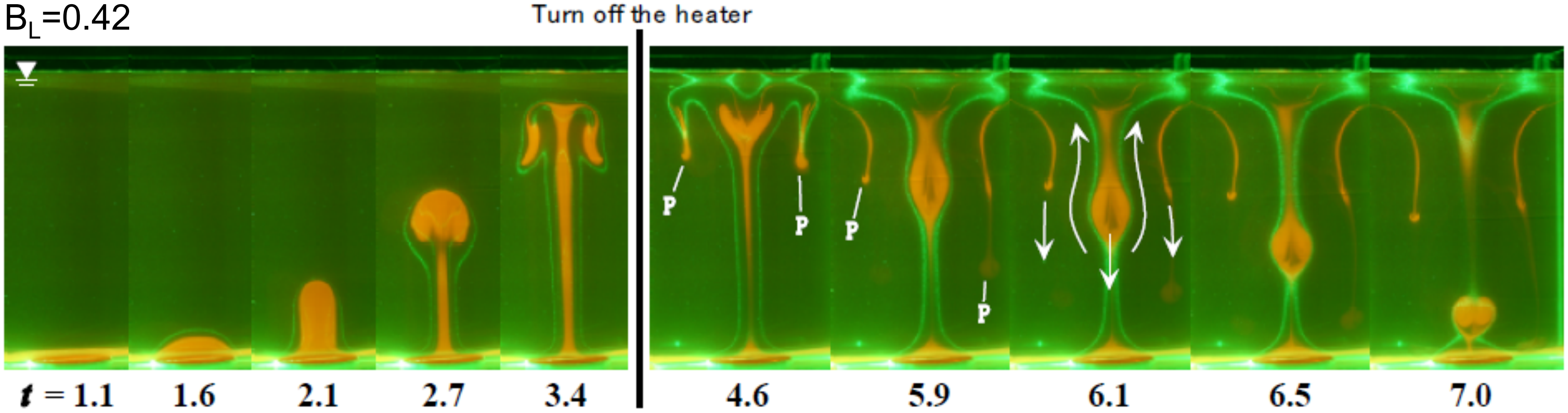
(Kumagai, Davaille & Kurita, 2007)

Localized thermochemical plumes: Time-dependence

B_L=0.59



B_L=0.42



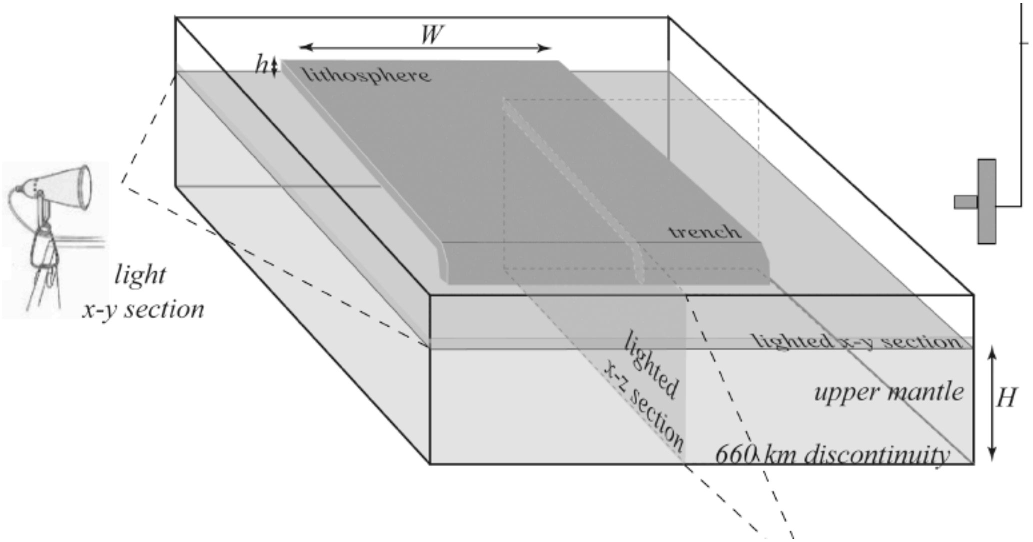
All thermo-chemical plumes should "fail" because of cooling!!

Failing Plume

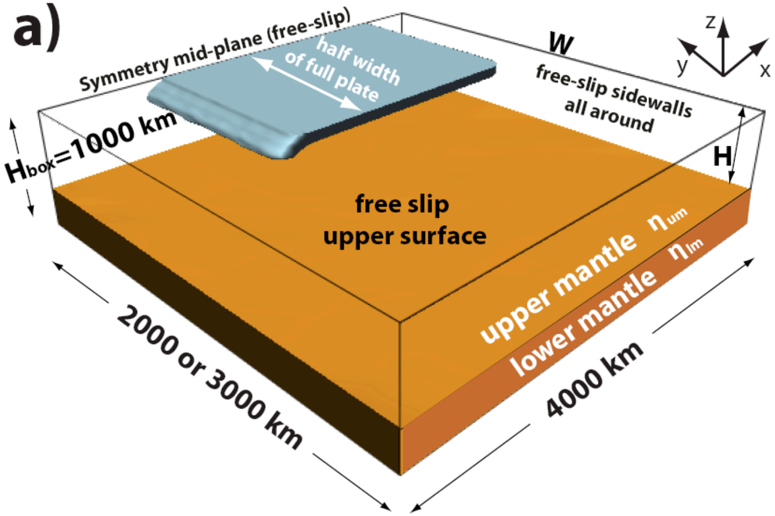
Regimes of Free Subduction

Models of free subduction

Experimental
(Funiciello et al. 2006)

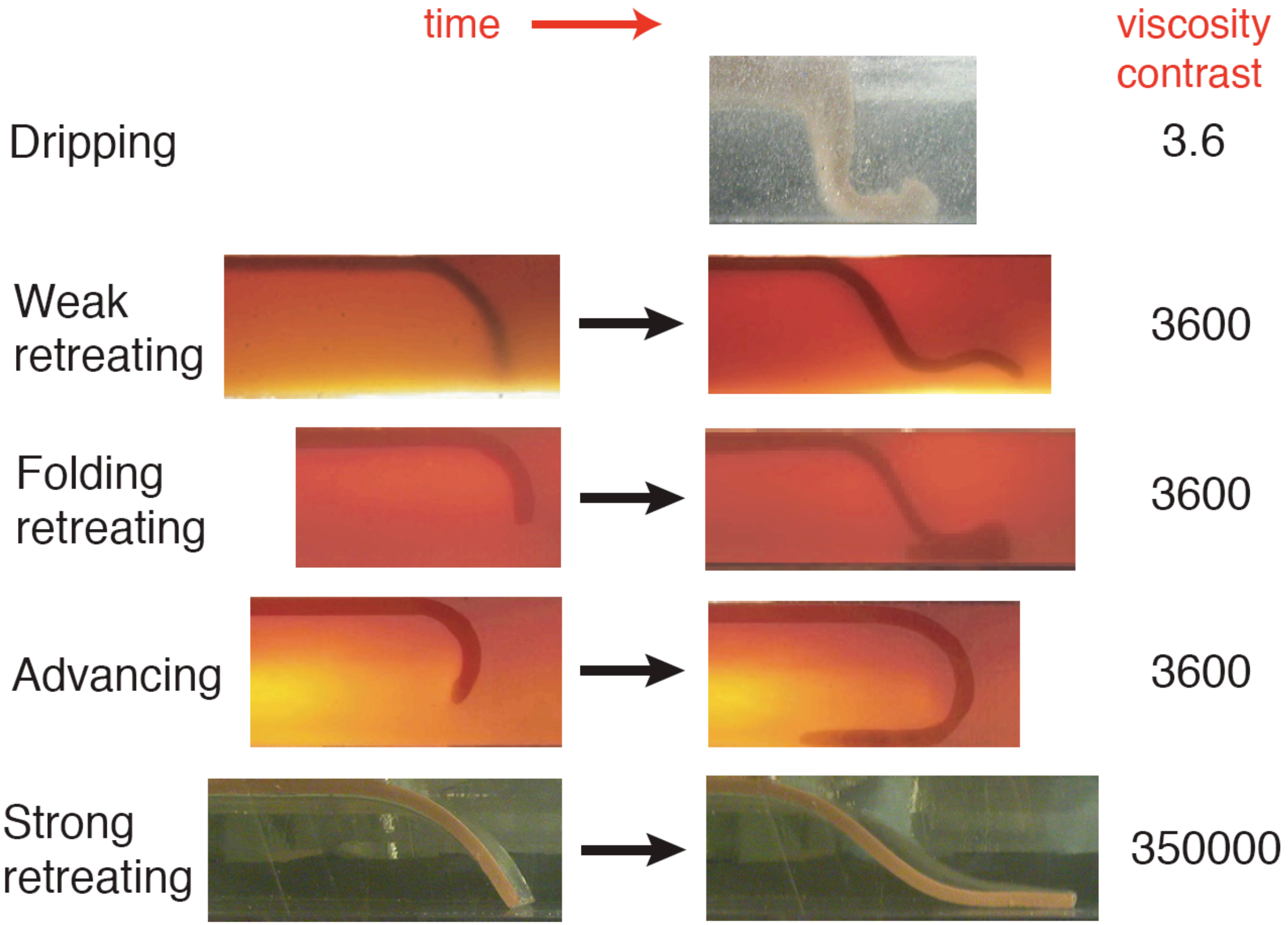


Numerical
(Stegman et al. 2009)



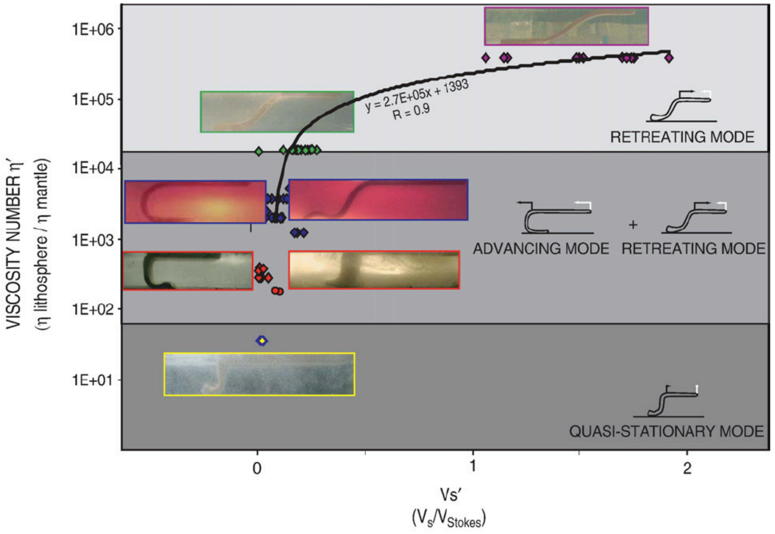
Modes of Free Subduction Observed in Laboratory Experiments

(Experiments from Roma-TRE, Monash, …)

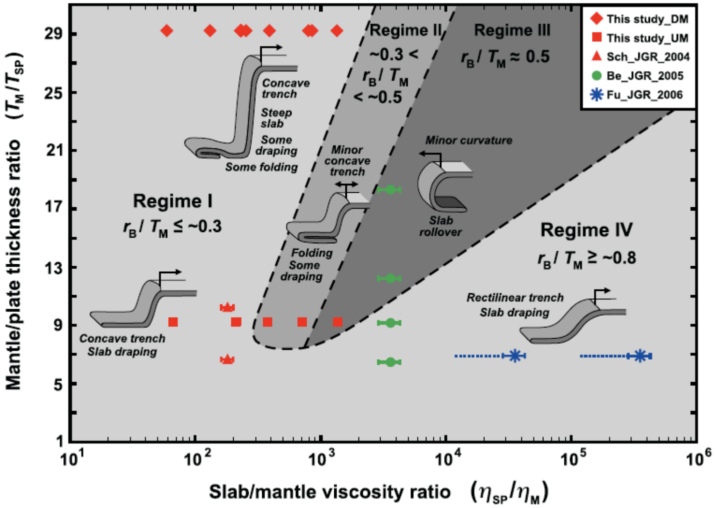


Regime diagrams for subduction modes

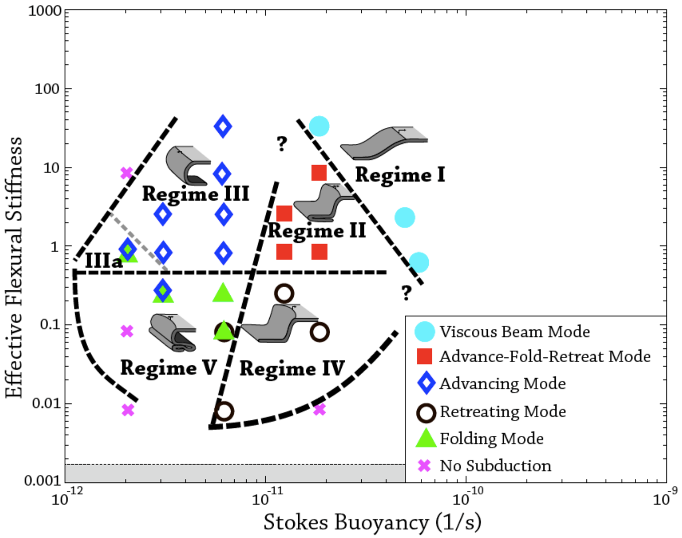
Funiciello et al. (2008)
(experimental)



Schellart (2008) (experimental)

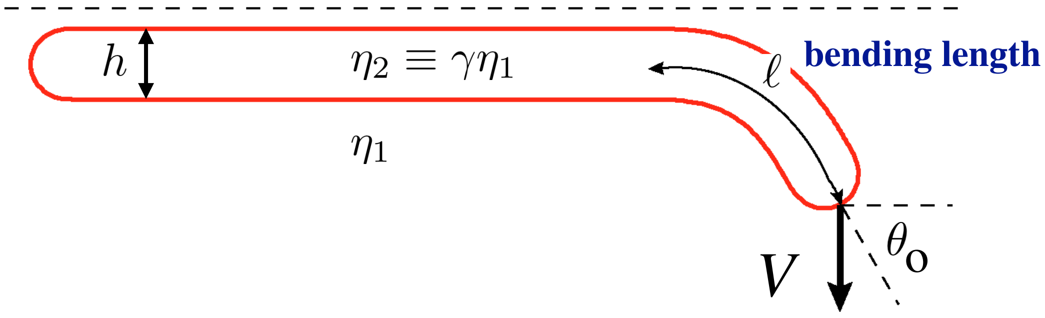


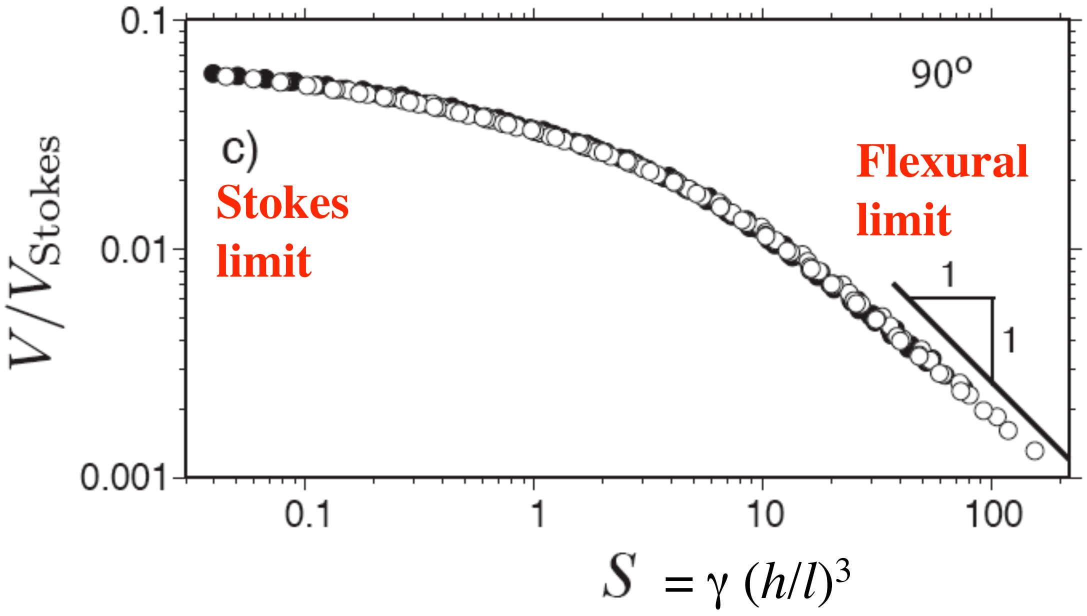
Stegman et al. (2009) (numerical)



Universal scaling law for slab sinking speed V

(Ribe 2009)

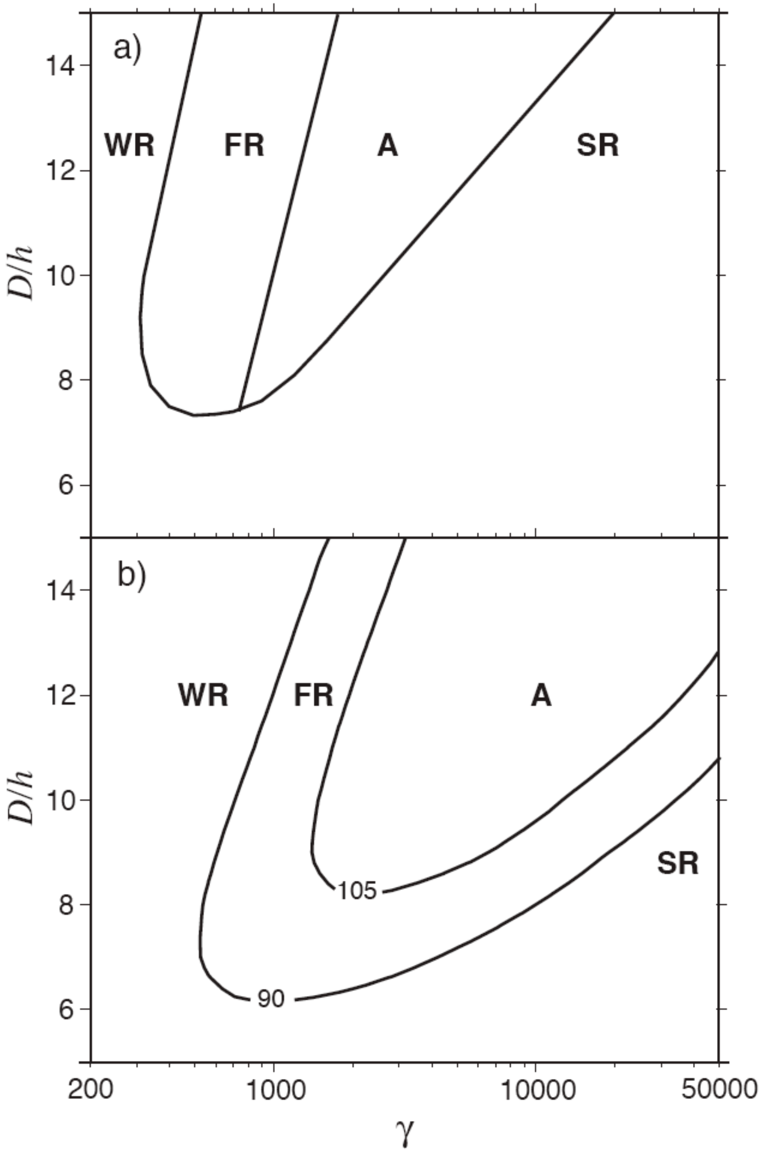




(188 numerical solutions)

Regime diagram for subduction modes: Experimental vs. numerical

Layer/plate thickness ratio



viscosity contrast

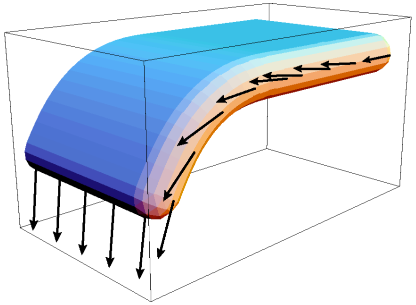
Schellart (2008)
(3D experimental)

Ribe (2009)
(2D numerical;
contours = dip of the slab
when it reaches the bottom)

ER2: Analytical and Numerical Models of Free Subduction

Neil M. Ribe (FAST); Claudio Faccenna (Roma-Tre)
Collaborators: P. Tackley, J. Wookey, H. Paulssen

- **Motivation:** to understand the physical origin of different free subduction modes



- **Approach:** 3D boundary-element numerical method + asymptotic thin-sheet theory

- **Goals:**

1. Predictive scaling laws/phase diagrams for key subduction parameters (slab morphology, plate speed, speed of trench retreat/advance, seismic anisotropy, state of stress in the slab …)
2. Comparison with laboratory experiments (Roma-Tre)
3. Comparison with geophysical observations